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Mechatronic Control Model of an inverted pendulum

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Abstract: *The inverted pendulum, a popular mechatronic application, is considered as a special class of unstable, non-linear, multivariable and complex mechatronic systems with two degrees of freedom and a single control input. This system is considered as a keen area of interest for researchers in the field of stabilization, control engineering and robotics. In this paper, a mechatronic approach to design a controller for an inverted pendulum is presented through a Bong Graph Method. First, a non-linear dynamic model of the inverted pendulum is developed by means of the Bond Graph Approach. Second, the proposed control law is derived from the Inverse Bond Graph Model of the inverted pendulum using the Bicausality concept. The robustness and effectiveness of the proposed control is verified and simulation results are conducted so as to confirm the validity of the proposed technique. Hands-on experience is carried out by means of the 20-sim software package (a demo version is freely available on the Internet).*

Keywords: *Bicausality,Bond Graph,Mechatronic, Inverted Pendulum Systems, Inverse Model, PID Control Design, Stability.*

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I. Introduction

Mechatronic design requires that a mechanical system and its control system be designed as an integrated system [1,2]. In order to make proper choices early in the design stage, tools that support modelling and simulation of physical systems are required - together with the controllers - with parameters which are directly related to the real-world system. The inverted pendulum is a popular mechatronic application that exists in many different forms. The common thread among these systems is their goal: to balance a link (that has its center of mass above its pivot point) on end using feedback control. This can be done either by applying a torque at the pivot point, by moving the pivot point horizontally as part of a feedback system, changing the rate of rotation of a mass mounted on the pendulum on an axis parallel to the pivot axis and thereby generating a net torque on the pendulum. The inverted pendulum has been employed in various devices and trying to balance an inverted pendulum presents a unique engineering problem for researchers. The inverted pendulum was a central component in the design of several early Seismometers due to its inherent instability resulting in a measurable response to any disturbance. The inverted pendulum model has been used in some forms of personal transportation devices [3,4,5]. Two-wheeled wheel chairs and other two-wheeled motorized vehicles can offer enhanced mobility for the driver.

A vast range of contributions exists for the control of different types of inverted pendulums. In [6] a stabilization and control of inverted pendulum on cart moving on an inclined surface using PID and fuzzy controllers was studied. The PID gains were obtained using trial and error method. The results showed better performance of PID controller over the other controller (the fuzzy one). The authors of [7] developed a hybrid fuzzy control strategy for two-wheeled robotic vehicle having a movable payload. The system was designed to move in different environments and terrains. The Euler-Lagrange approach was used for deriving a model of the system. The paper [8] discussed the implementation of an event-based control structure for the classical rotary inverted pendulum, using a real shared communication medium to close the loop. The objective was to reduce the amount of information exchanged between the controller and the plant without a significant loss of control performance. The results show how the threshold-based communication can be easily used to significantly reduce the consumed bandwidth; and the behavior of the system was almost the same as the one with conventional control. In [9] a sliding-mode control for tracking control and stabilization of X–Z inverted pendulum was applied. Its performance was compared with that of the PID control. Simulation results feature that the design scheme of sliding-mode control is more efficient for the stabilization and tracking control of the X–Z inverted pendulum. Authors of [10] presented a fractional PI-state feedback controller design for controlling and stabilization of an inverted pendulum-cart system. It was based upon choosing n-1 poles of an integer polynomial and decomposing a fractional polynomial into a first order. The proposed control efficiency is examined through experiments. In [11] a constructive method to design a controller for an inverted pendulum characterized by a time-delayed balance control was presented. The control design was based on a nonlinear state predictor scheme. The contribution [12] reported a first attempt on analyzing the effect of time delay on the stability of a planar pendulum, where the author used the PMD (Proportional Minus Delay controller) technique to provide sufficient conditions for the stability of the inverted pendulum when the control input action is delayed. Authors of [13] developed a control strategy for the stabilization of a flywheel inverted pendulum with minimum energy. The controller was designed basically in two control loops. First, an output stabilizer PID controller is obtained following a traditional control design. The appearance of internal instability requirements for a second control loop, was also designed by a simple control design method. The work [14] developed an adaptive output recurrent cerebellar-model-articulation-controller (AORCMAC) for angle and position control of such a wheeled inverted pendulum without model information.

Our paper introduces a mechatronic modeling and control of an inverted pendulum. Initially, the Bond Graph tool is used to model the inverted pendulum. Then, we discuss the theoretical development related to the control of the inverted pendulum using the concept of Bond Graph Bicausality. This method can represent the whole system (model and control) and features some properties that can be directly applied to the model. Finally, the performance of the system is compared with a conventional PID controller to validate it. A Bond Graph [15,16,17] is a graphical representation of a physical dynamic system. It is similar to the better known block diagram and signal-flow graph, with the major difference that the arcs in bond graphs represent bi-directional exchange of physical energy, while those in block diagrams and signal-flow graphs represent unidirectional flow of information. Also, bond graphs are multi-energy domain (e.g. mechanical, electrical, hydraulic, etc.). This means that a Bond Graph can incorporate multiple domains seamlessly.

The Bond Graph is composed of the "bonds" which link together "single port", "double port" and "multi-port" elements (*R, I, C, TF* and *GY*) [15,16,17]. Each bond represents the instantaneous flow of energy (*dE(t)/dt*) or power *P(t)*. A pair of variables called "power variables" whose product is the instantaneous power of the bond denotes the flow of energy in each bond. Each domain's power variables are broken into two types: "*effort e(t)*" and "*flow f(t)*". Effort multiplied by flow produces power, thus the term power variables. Every domain has a pair of power variables with corresponding effort and flow variables. Causality - a bond graph must determine which of the two power signals for the subsystem is entering, and which in turn dependent variable, thus acting on the subsystem. Causality is referred by perpendicular to the detention site where the flow enters the subsystem variable effort. Between the building elements of bond graphs which in practice are sufficient, and that we classify according to the number of bonds are one-port, two-port and multiport.

One-ports are elements that exchange energy in the system only via one link. This group includes:

- Source of effort "SE".
- Source of flow "SF".
- One port C element (Capacitor).
- One port I element (Inductor).
- One port R element (Resistor).

Source of effort - the ideal source type of effort is maintained on a constant effort level. The flow f is given by connected load and for ideal cases immediately affect the value of effort is not influenced. Source of flow - the flow is maintained either at a constant level or as a function of time. The effort is intended load applied and, ideally, does not affect the instantaneous flow value.

For non-ideal effort and flow sources this are not so (Fig. 1):

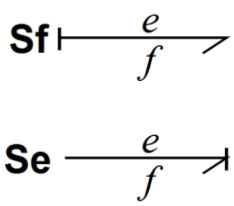

Fig.1 Symbolic sign of flow and effort sources

Two-ports are elements of the system which can exchange energy via two bonds. Thus, two-ports retain power, it is supposed that the product of effort and flow at the exit is equal to the product of effort and flow at the inlet. There are two basic types of two-ports:

- Transformer "TF".
- Gyrator "GY".

One-ports are attached to two-ports in the bond graph by connecting nodes. The power is branched in the nodes. There are two types of nodes:

- 1 – junction.
- 0 – junction.

The 1 – junction for all the power bonds that lead to the same node flow (f) and node describes the balance of effort (e). 0 - junction is the power of all bonds that lead to the same node of effort (e) and node describes the balance of the flow (f). The main advantages of the Bond Graph tool for modeling purposes are summarized through few keywords:

1. Modeling: the Bond Graph is a unified representation language, which explicitly highlights the power flows, makes possible the energetic study, simplifies models building for multi-disciplinary systems, explicitly shows up the cause - effect relations (causality) and leads to a systematic writing of mathematical models (linear or nonlinear associated).
2. Identification: identification of unknown parameters, but knowledge of the associated physical phenomena and mastering physical meaning of the obtained model.
3. Analysis: Putting to the fore the causality problems, and therefore the numerical problems, model dynamic estimation and identification of the slow and fast variables.
4. Control: Design of control laws from simplified models.
5. Simulation: Specific software (20-Sim)

In this contribution, the Bond Graph is used to model the inverted pendulum and the control law is obtained by means of bicausality concept [18].

II. Bond Graph Model Of The Inverted Pendulum

The inverted pendulum system depicted in Fig. 2 consists of a link mounted on a cart by means of a pivot in such a way that the pole can freely swing in the (*xz*) plane.

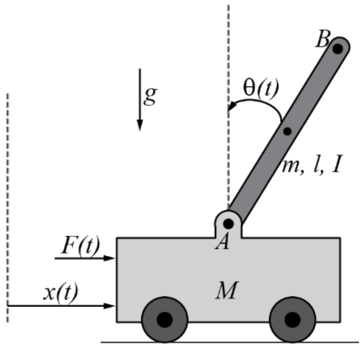

Fig.2 Schematic representation of an inverted pendulum on a cart.

The following variables and parameters have been chosen to describe the system:

- x is the position of the cart
- (x_p, y_p) denote the position of the pendulum
- θ the angular position of the pendulum
- M is the mass of the cart
- m is the mass of the pendulum
- l is half the length of the pendulum, i.e., the distance from the pivot (point A) to the center of mass of the pendulum.
- I is the pendulum's moment of inertia
- b_1 is the friction between the car and the ground
- b_2 is the friction in the pivot
- g is the gravity acceleration
- *F(t)* is the horizontal force being applied to the cart to swing the pendulum until it reaches the desired unstable position.

The cart-pole system has two equilibrium points, one of which is known as the stable vertically downward position where $\theta = \pi$ and the other one being the unstable vertically upward position where $\theta = 0$. The relationships between velocities are:

$$\acute{x}_p = \acute{x} + l\acute{\theta}cos\theta \quad (1)$$

$$\acute{y}_p = -l\acute{\theta}sin\theta \quad (2)$$

From Fig. 1 and Eq. (1) and (2) we can directly build the bond graph model sketched in Fig. 3.

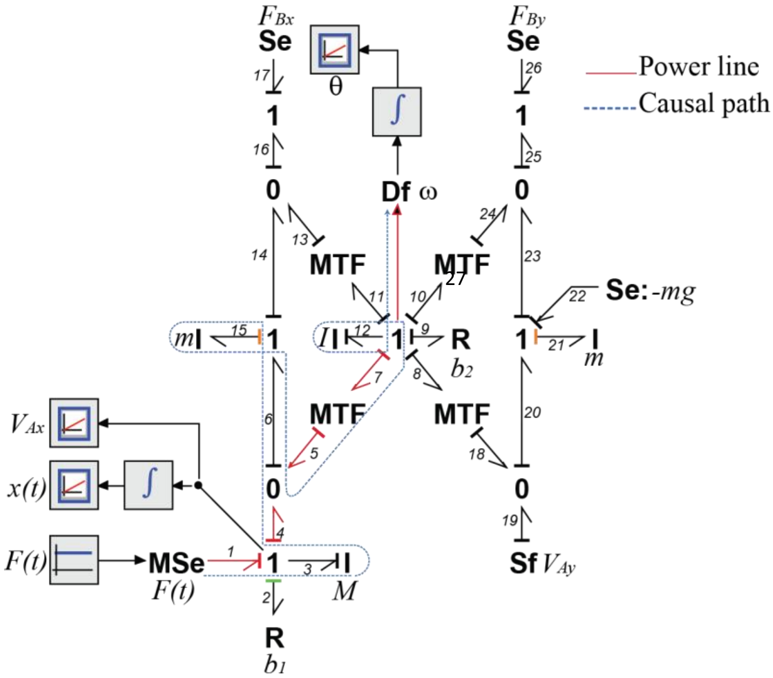

Fig.3 Bond Graph Model of an inverted pendulum.

The masses of the pendulum and the car and the pendulum's inertia are represented by I-element. The friction in the pivot and between the car and the ground are modeled by R-element. The velocities in the system are represented by 1-jonction and the kinematic relations between velocities are modeled using MTF-element and 0-jonction. The boundary condition on the top of the pendulum (point B) is represented by effort source (Se-element) equal to zero ($F_{Bx} = F_{By} = 0$) because the top of the pendulum is free. The boundary condition on the bottom of the pendulum is represented by flow source (Sf-element) equal to zero in the y-direction because the car does not move in the y-direction. The horizontal force *F(t)* applied to the cart is modeled by a modulated effort source (MSe-element), the modulated element being used because the applied force is variable.

III. Control Of Inverted Pendulum Using The Inverse Bond Graph Model.

From the bond graph model, it can be seen that the pendulum angle is influenced by the horizontal force action *F(t)*. Therefor the control strategy presented in this paper adjusts the horizontal force *F(t)* to set the pendulum at its equilibrium position. In order to control the system, it is necessary to generate the reference horizontal force. To this aim, a specific algorithm is designed, based upon the Inverse Bond Graph (IBG) [18] and the performance of the system is compared with a conventional PID controller to validate it. The inverse model corresponds to a re-organization of the equations where the input and output roles are exchanged: inputs become outputs and vice versa [19]. The inverse model is created by imposing both effort and flow information from the sensor and receiving both at the source. This procedure cannot be done through normal causality. That's why the notion of bicausality [20, 21] is introduced. This is graphically represented by decomposing the causal trait into two half-causal traits.

For the inverse bond graph formulation, it is necessary to change the flow detectors (*Df:θ(t)*), which will be placed in bond 27 (Fig. 3) of the original bond graph, by a source named *SS*, (which impose zero effort but non-zero flow to the inverse model). Then the bicausality propagates (in only one line of power transfer) from this source (*SS:θ(t)*) to the input effort source (*Se: F(t)*) of the original bond graph which becomes a detector (i.e. *SS:F(t)*) in the inverse bond graph. Direct bond graph model analysis (Fig. 3) indicates that there is a power line and a causal path between the input variable *F(t)* and the output variable*θ(t)*. Therefore, the model is structurally invertible [21] compared to these couples of variables *F(t)* and*θ(t)*. The controller objective is to calculate the horizontal force *F(t)* required to set the pendulum at its equilibrium position. It is then appropriate to inverse the bond graph model of Fig. 2 relatively to the couples of variables *F(t)* and*θ(t)*.

The Inverse Bond Graph model of the system is given in Fig. 4. From this model we can derive a block diagram model, which is shown in Fig. 5, to control the pendulum angle in an open loop, the closed loop control is done by fixing the error dynamics. The proposed gains of the error dynamics are PI-controllers; there estimate values are considered in the control law.

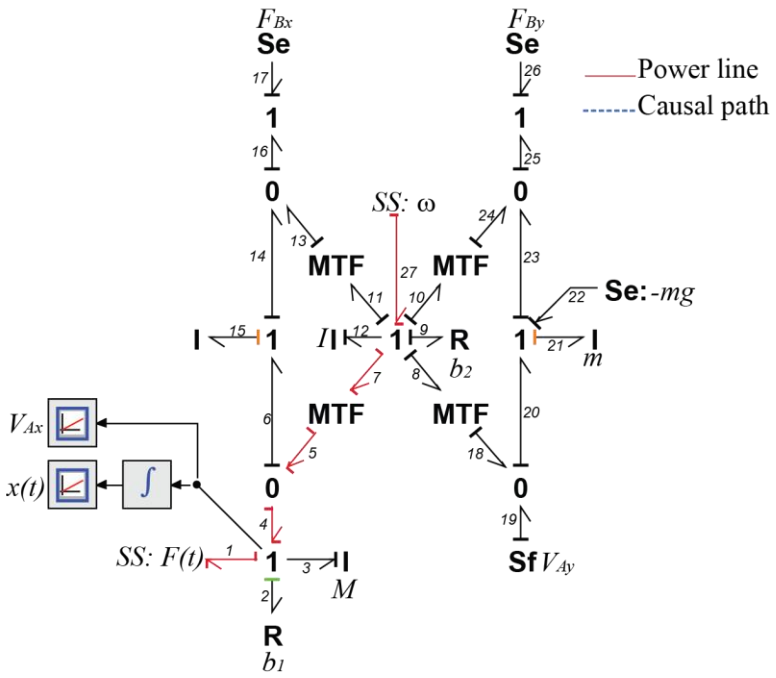

Fig. 4 Inverse Bond Graph Model of an inverted pendulum.

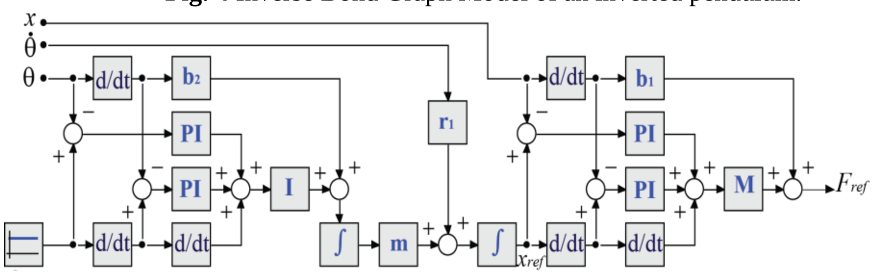

Fig. 5 Inverse Bond Graph Model control law block diagram

IV. Results And Discussion

In order to verify the robustness of the proposed control law, a simulation is carried-out on 20-Sim framework, considering the following system parameters $M=0.2$ kg, $m=0.1$ kg, $I=0.006$ kg.m^2, $b_1=0.005$ kg.s^{-1}, $b_2=0.005$ kg.m^2.s^{-1} and $l=0.1$ m.

The initial condition for the simulation is chosen as *0* for all state variables except the angle of the pendulum, whose initial conditions are: $\pi/6$, $\pi/5$, $\pi/4$, $\pi/3$.

To show the efficiency of proposed method in this paper, the Inverse Bond Graph control design is compared with the PID controller that are proposed in [22]. The structure of the control method with Inverse Bond Graph control and PID control is given in Fig. 6 and 7 respectively. The PID parameters are designed as follows:

$$PID_\theta\text{: P} = 25,\ \text{I} = 15,\ \text{D} = 3.$$
$$PID_x\text{: P} = -1.5,\ \text{I} = -0.5,\ \text{D} = -0.2.$$

The simulation results are represented in Fig .8, which shows the pendulum angle and car position for 4 initial conditions of pendulum angle. The results showed that both IBG and PID controllers can maintain the stability of the pendulum in its unstable equilibrium position for all initial conditions. It is clear that with the IBG controller, the pendulum vibrations (for both angle and position) are successfully reduced compared with PID controller.

Comparing simulation results of Fig. 8, we can find that stabilization of the inverted pendulum with the IBG controller has better performance than that of with PID control. The IBG controller not only has faster response speed, but also has less stable error.

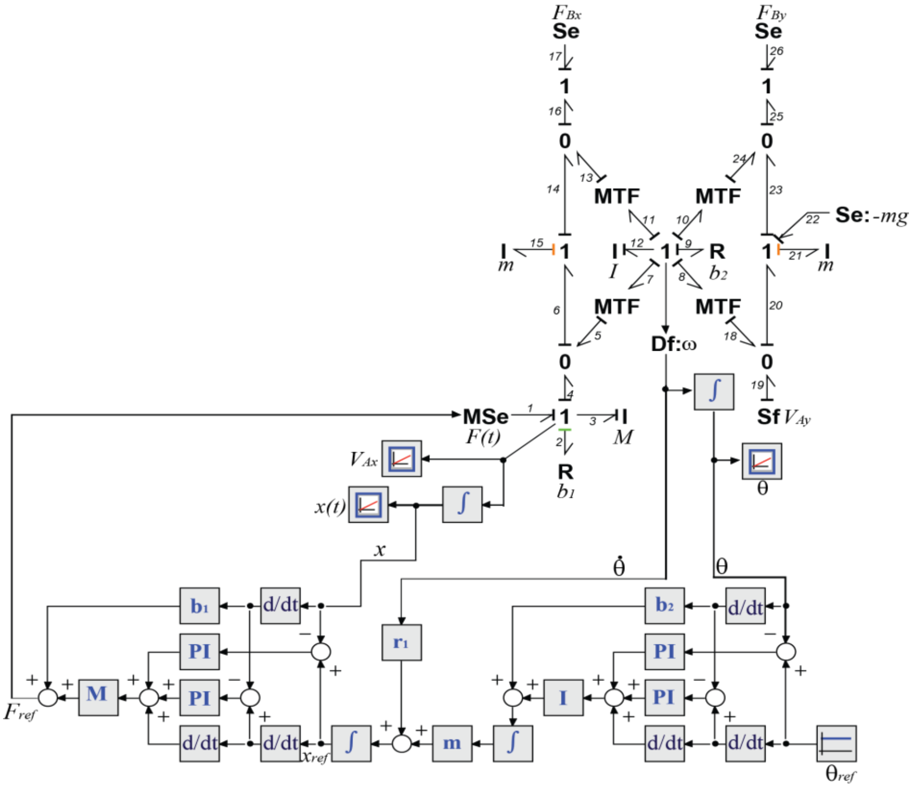

Fig.5 Control structure of IBG.

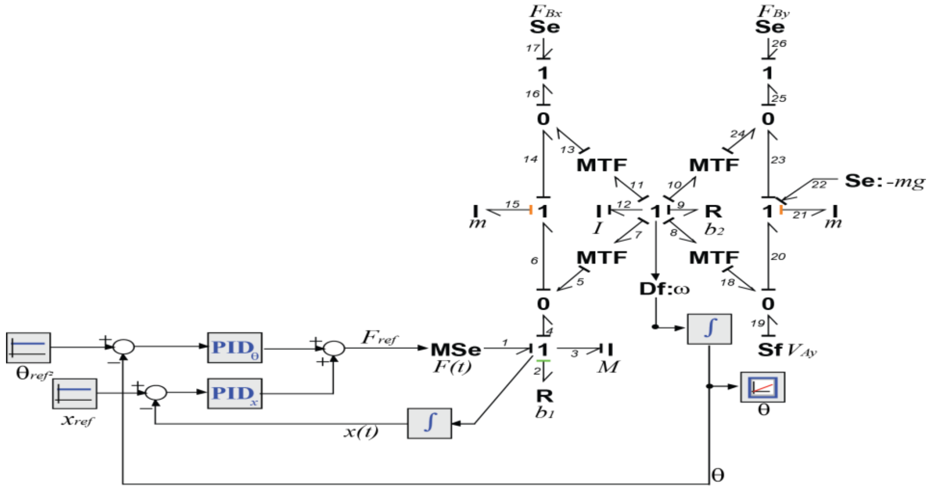

Fig.4 Control structure of PID.

V. Conclusion

In this paper, we have proposed a mechatronic approach to design a controller for an inverted pendulum through a Bong Graph Method. First, the model of the inverted pendulum is developed by means of the Bond Graph Approach. Then, the proposed control law is derived from the Inverse Bond Graph Model of the inverted pendulum using the Bicausality concept. The proposed technique was compared with a PID control strategy, the simulation results showed better performance of Inverse Bond Graph controller over PID controller.

References

[1]. Breedveld Peter C. Port-based modeling of mechatronic systems. Math Com-put Simul 2004;66:99–127.
[2]. Buur J. A theoretical approach to mechatronics design [Ph.D. dissertation];1990.
[3]. J.H. Lee, H.J. Shin, S.J. Lee, S. Jung. Balancing control of a single-wheel inverted pendulum system using air blowers: Evolution of Mechatronics capstone design. Mechatronics 23 (2013) 926–932.
[4]. Hyungjik Lee, Seul Jung. Balancing and navigation control of a mobile inverted pendulum robot using sensor fusion of low cost sensors. Mechatronics 22 (2012) 95–105.
[5]. Chih-Hui Chiu, You-Wei Lin, Chun-Hsien Lin. Real-time control of a wheeled inverted pendulum based on an intelligent model free controller. Mechatronics 21 (2011) 523–533.
[6]. Ashwani Kharola, Pravin Patil, Suyashi Raiwani, Deepak Rajput. A comparison study for control and stabilization of inverted pendulum on inclined surface (IPIS) using PID and fuzzy controllers. Perspectives in Science (2016) 8, 187–190.
[7]. Almeshal, A.M., PhD Thesis submitted to Dept. of Automobile Control and System Engineering 2013. Development and control of a novel-structure two-wheeled robotic vehicle maneuverable indifferent terrains. University of Sheffield, UK.
[8]. Casanova V, et al. Control of the rotary inverted pendulum through threshold-based communication. ISA Transactions (2016), http://dx.doi.org/10.1016/j.isatra.2016.01.009i
[9]. Jia-Jun Wang. Stabilization and tracking control of X–Z inverted pendulum with sliding-mode control. ISA Transactions 51 (2012) 763–770.
[10]. M. Bettayeb, C. Boussalem, R. Mansouri, U. M. Al-Saggaf. Stabilization of an inverted pendulum-cart system by fractional PI-state feedback. ISA Transactions 53 (2014) 508–516.
[11]. Ghommam J, Mnif F. Predictor-based control for an inverted pendulum subject to networked time delay. ISA Transactions (2017), http://dx.doi.org/10.1016/j.isatra.2017.01.015i
[12]. Atay FM. Balancing the inverted pendulum using position feedback. Appl Math Lett 1999;12:51-6.
[1] Olivares M, Albertos P. Linear control of the flywheel inverted pendulum. ISA Transactions (2014), http://dx. doi.org/10.1016/j.isatra.2013.12.030i
[2] Chih-Hui Chiu, You-Wei Lin, Chun-Hsien Lin, Real-time control of a wheeled inverted pendulum based on an intelligent model free controller. Mechatronics 21 (2011) 523–533
[3] Margolis Donald. Bond Graph Modelling of Engineering Systems: Theory,Applications and Software Support; 2011.
[4] Merzouki Rochdi, Samantaray Arun Kumar, Pathak Pushparaj Mani, Ould Bouamama Belkacem. Intelligent mechatronic systems: modeling, control and diagnosis. 2013.
[5] Zakaria Khaouch, Mustapha Zekraoui, Jamaa Bengourram, Nourreeddine Kouider, Mustapha Mabrouki. Mechatronic modeling of a 750 kW fixed-speed wind energy conversion system using the Bond Graph Approach. ISA Transactions 65 (2016) 418436.
[6] P.J. Gawthrop, Bicausal bond graph, in: Proceeding of the International Conference on Bond Graph Modeling and Simulation ICBGM95, vol. 27, 1995, pp. 8388.
[7] Gawthrop P.J. 2000. Physical interpretation of inverse dynamics using bicausal bond graphs. J. Frankl. Inst., 337(6): 743–69.
[8] Ngwompo R.F., Scavarda S. 1999.Dimensioning problems in system design using bicausal bond graphs. Simul. Pract. Theory7, 577–587.
[9] Ngwompo R.F., Scavarda S., Thomasset D. 2001. Physical model-based inversion in control systems design using bond graph representation—part 2: applications. Proc. IMechE Part I J. Syst. Control Eng, 215: 105–12.
[10] Wang JJ. Simulation studies of inverted pendulum based on PID controllers. Simulation Modelling Practice and Theory 2011;19(1):440–9.

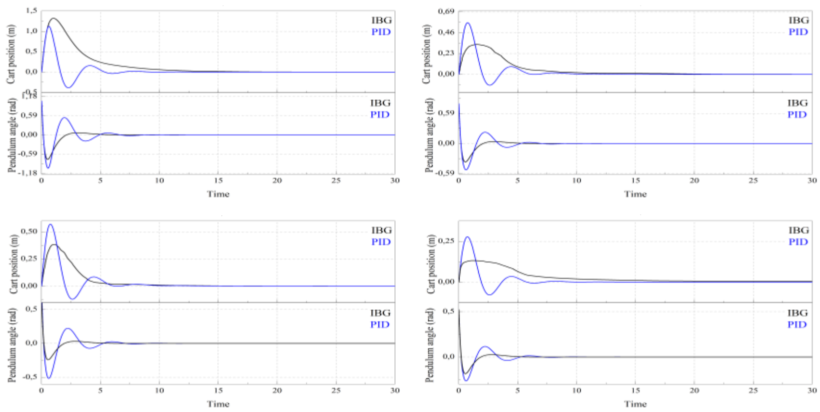

Fig.6 Simulation response of the cart position and pendulum angle for

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